MATH 108 Fall 2019 - **Problem Set 2**

due October 11

1. Let x and y be real numbers.
   (a) Prove for all x and y that if $x + y$ is irrational then x is irrational or y is irrational.
   (b) Prove for all x that there exists y such that $x + y$ is rational.
2. For all integers x, prove that x is divisible by 6 if and only if x is divisible by 2 and by 3.
3. (a) Prove that there exist integers m and n such that $3m + 4n = 1$.
   (b) Prove that there does not exist integers m and n such that $3m + 6n = 1$.
4. Let $A = \{1, 2\}$ and $B = \{1, 4, 5\}$.
   (a) Find $A \cup B$.
   (b) Find $A \cap B$.
   (c) Find $A \setminus B$.
   (d) Find $A \times B$.
   (e) Find $\mathcal{P}(A)$.
5. Let A, B, C, D be sets. Prove the following propositions.
   (a) $(A \cup B) \cap C \subseteq A \cup (B \cap C)$.
   (b) $(A \setminus B) \cap (A \setminus C) = A \setminus (B \cup C)$.
   (c) If A and B are disjoint, then $A \cap C$ and $B \cap C$ are disjoint.
   (d) If $C \subseteq A$ and $D \subseteq B$ then $D \setminus A \subseteq B \setminus C$.
6. Let A be the set of positive integers that are not perfect squares. Let P be the set of prime numbers. Prove that $P \subseteq A$.
7. Let S be a set of 4 distinct integers. Prove that there exists a pair of distinct elements $x, y \in S$ such that $x - y$ is divisible by 3.