MATH 108 Fall 2019 - **Problem Set 7**

due November 15

1. For each function f, determine if it is surjective. If yes, find a *right-inverse* of f, which is a function g such that $f \circ g$ is the identity.
   (a) $f : \mathbb{R} \to \mathbb{R}^2$ defined by $f(x) = (x, x)$.
   (b) $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = x + y$.
   (c) $f : \mathbb{Z} \to \mathbb{Z}/4\mathbb{Z}$ defined by $f(x) = \overline{x}$.
   (d) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^x$.
   (e) $f : \mathbb{Z} \to \{0\}$ defined by $f(x) = 0$.
2. Let $f : A \to B$ and $g : B \to C$.
   (a) Prove that if $g \circ f$ is surjective then g is surjective.
   (b) Give an example of f and g where $g \circ f$ is surjective but f is not surjective.
3. Prove that each function is a bijection. Give the inverse.
   (a) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = x + 1$.
   (b) $f : (2, \infty) \to (-\infty, -1)$ defined by $f(x) = \dfrac{-x}{x-2}$.
   (c) $f : \mathbb{Z}/8\mathbb{Z} \to \mathbb{Z}/8\mathbb{Z}$ defined by $f(\overline{x}) = \overline{5x - 1}$.
4. For each pair of sets, find a bijection from the first to the second.
   (a) $\mathbb{Z}_{>0}$ and $\mathbb{Z}_{\geq 0}$.
   (b) $\mathbb{R}^2$ and $\mathbb{C}$.
   (c) $\mathbb{Z}$ and $\mathbb{Z}_{>0}$.
   (d) $\{x \in \mathbb{R} \mid -1 < x < 1\}$ and $\mathbb{R}$.
5. For postive integers n and m, let $[n] = \{1, 2, \ldots, n\}$ and $[m] = \{1, 2, \ldots, m\}$.
   (a) Let A be the set of all functions from $[n]$ to $[m]$. Compute $|A|$ in terms of n and m.
   (b) Let B be the set of all bijective functions from $[n]$ to $[m]$. Compute $|B|$ in terms of n and m.
   (c) Let C be the set of all injective functions from $[n]$ to $[m]$. Compute $|C|$ in terms of n and m.
6. Let $f_1, f_2 : A \to B$ and $g : B \to C$ and $h_1, h_2 : C \to D$.
   (a) Prove that if $g \circ f_1 = g \circ f_2$ and g is injective, then $f_1 = f_2$.
   (b) Prove that if $h_1 \circ g = h_2 \circ g$ and g is surjective, then $h_1 = h_2$.