MATH 108, **Final Exam**, March 22, 2019

NAME:

No notes or calculators are allowed. Show all your work.

problem	1	2	3	4	5	6	7	8	9	total
points										
maximum	5	5	4	4	6	5	6	5	5	45

1. Prove or disprove that the following set is a subgroup of $(\mathbb{R}^2, +)$: [/5]

$$\{(0, y) \mid y \in \mathbb{R} \text{ and } y \geq 0\}.$$

2. Let A be the set of functions from $\mathbb{N}_1$ to $\mathbb{Z}/3\mathbb{Z}$. Prove that A is uncountable. [/5]

3. Determine whether $3^{100} + 1$ is divisible by 4. (Please show your work!) [/4]

4. Find the cardinality of the set of all **non**-injective functions from $\{1, 2\}$ to $\{1, 2, 3, 4\}$. [/4]

5. Let $f : \mathbb{Z}^2 \to \mathbb{Z}$ defined by $f(x, y) = 1 + x + y$.

(a)
- Is f injective? **Yes** or **No** (circle one) [/2]
- Is f surjective? **Yes** or **No** (circle one)

(b) If f is injective, find a left-inverse. If f is surjective, find a right-inverse. [/4]

6. Let S be a set with partial order $\sqsubseteq$ and T be a set with partial order $\preceq$. Let $f : S \to T$ be an *order-embedding* function, meaning that $x \sqsubseteq y$ if and only if $f(x) \preceq f(y)$. Prove that f is injective. [/5]

7. Let G be the group $(\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}, +)$. The group operation is defined as

$$(a, b) + (c, d) = (a + c, b + d).$$

(a) Find the order of $(\overline{1}, \overline{1})$ and the order of $(\overline{2}, \overline{0})$. [/4]

(b)
- Is G abelian? **Yes** or **No** (circle one) [/2]
- Is G cyclic? **Yes** or **No** (circle one)

8. Find the cardinality of $\mathbb{R} \setminus \mathbb{Z}$ and prove your answer. [/5]

9. Let G and H be groups and let $f : G \to H$ be a group homomorphism. Prove that if G is abelian and f is surjective, then H is abelian. [/5]