# MATH 150A Winter 2020 - **Problem Set 9**

due March 13

1. Let $m$ be an orientation-reversing isometry of $\mathbb{R}^2$. Prove algebraically that $m^2$ is a translation.

2. Find the conjugacy class of an isometry of $\mathbb{R}^2$ of each of the following types.

   (a) Translation.

   (b) Rotation about a point.

   (c) Reflection across a line.

   (d) Glide reflection across a line.

3. Let $\ell_1$ and $\ell_2$ be lines through the origin in $\mathbb{R}^2$ that intersect at an angle of $\pi/n$ and let $r_i$ be the reflection across $\ell_i$. Prove that $r_1$ and $r_2$ generate a dihedral group $D_n$.

4. Let $S$ and $S'$ be subsets of $\mathbb{R}^n$. $S$ is *dense* in $S'$ if for every point $a \in S'$ and every $\varepsilon > 0$, there is $s \in S$ with $|a - s| < \varepsilon$.

   (a) Prove that an additive subgroup $G$ of $\mathbb{R}$ is either dense in $\mathbb{R}$ or else discrete.

   (b) Prove that the additive subgroup of $\mathbb{R}$ generated by 1 and $\sqrt{2}$ is dense in $\mathbb{R}$.

   (c) Let $H$ be a subgroup of $SO_2$. Prove that either $H$ is cyclic or dense in $SO_2$.

5. Find the symmetry group of

   (a) an I-beam, which one can think of as the product set of the letter I and an interval.

   (b) a baseball (or equivalently a tennis ball) accounting for the seam.