MATH 108 Fall 2019 - **Problem Set 5**

due November 4

1. Let $\sim$ be the relation on $\mathbb{R}$ defined by $x \sim y$ if and only if $x - y \in \mathbb{Z}$.
   (a) Prove that $\sim$ is an equivalence relation.
   (b) Prove for all real numbers x, y, z, w that if $\overline{x} = \overline{z}$ and $\overline{y} = \overline{w}$ then $\overline{x+y} = \overline{z+w}$.

2. Using modular arithmetic, prove that for all postive integers n,
   (a) $10^n - 1$ is divisible by 3.
   (b) $n^4 + 2n^3 - n^2 - 2n$ is divisible by 4.
   (c) $1^n + 2^n + 3^n + 4^n$ is a multiple of 5 or one less than a multiple of 5.

3. The "Cancellation Law" for $\mathbb{Z}/m\mathbb{Z}$ is the statement: For all $x, y, z \in \mathbb{Z}$, if $xy \equiv xz \pmod{m}$ and $x \not\equiv 0 \pmod{m}$ then $y \equiv z \pmod{m}$.
   (a) Prove that if m is prime then the Cancellation Law for $\mathbb{Z}/m\mathbb{Z}$ is true.
   (b) Prove that if m is composite then the Cancellation Law for $\mathbb{Z}/m\mathbb{Z}$ is false.

4. Let $\preceq$ be the relation on $\mathbb{Z}^2$ defined by $(a, b) \preceq (c, d)$ if and only if $a \leq c$ and $b \leq d$.
   (a) Prove that $\preceq$ is a partial order.
   (b) Find the greatest lower bound of $\{(1, 5), (3, 3)\}$.
   (c) Is $\preceq$ a total order? Justify your answer.

5. Let A be the set of divisors of 36, $A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$. Draw the Hasse diagram for the poset $(A, |)$.