MATH 108 Fall 2019 - **Problem Set 6**

due November 8

1. For each pair of sets A and B, and subset $\Gamma \subseteq A \times B$ determine if Γ is the graph of a function from A to B. Justify your answer.
   (a) $A = B = \mathbb{R}$ and $\Gamma = \{(x, y) \in \mathbb{R}^2 \mid x = y^2\}$.
   (b) $A = B = \mathbb{R}$ and $\Gamma = \{(x, y) \in \mathbb{R}^2 \mid y = x^2\}$.
   (c) $A = B = \mathbb{R}$ and $\Gamma = \{(x, y) \in \mathbb{R}^2 \mid y = \sqrt{x}\}$.
   (d) $A = B = \mathbb{Z}$ and $\Gamma = \{(n, 0) \mid n \in \mathbb{Z}\}$.
   (e) $A = \mathbb{Z}$, $B = \{0\}$ and $\Gamma = \{(n, 0) \mid n \in \mathbb{Z}\}$.
   (f) $A = B = \mathbb{Z}/5\mathbb{Z}$ and $\Gamma = \{(a, b) \mid a = \overline{2}b\}$.

2. For each function f, determine if it is injective. If yes, find a *left-inverse* of f, which is a function g such that $g \circ f$ is the identity.
   (a) $f : \mathbb{R} \to \mathbb{R}^2$ defined by $f(x) = (x, x)$.
   (b) $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = x + y$.
   (c) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 2x$.
   (d) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^x$.
   (e) $f : \mathbb{Z} \to \{0\}$ defined by $f(x) = 0$.

3. Let $f : A \to B$ and $g : B \to C$.
   (a) Prove that if $g \circ f$ is injective then f is injective.
   (b) Give an example of f and g where $g \circ f$ is injective but g is not injective.

4. Let S be a set with partial order $\sqsubseteq$ and T be a set with partial order $\preceq$. A function $f : S \to T$ is called *order-embedding* if it satisfies the property that $x \sqsubseteq y$ if and only if $f(x) \preceq f(y)$. Prove that if f is order-embedding then f injective.

5. Let a and m be integers with $0 < a < m$. Let $f : \mathbb{Z}/m\mathbb{Z} \to \mathbb{Z}/m\mathbb{Z}$ be the function defined by $f(\overline{x}) = \overline{a} \cdot \overline{x}$. Prove that f is injective if and only if $\gcd(a, m) = 1$.