

MATH 108 Fall 2019: Intro to Abstract Math
Final topics

Chapters 1-3 (see midterm topics list)

Posets (Sec 3.5)

- Partial orders
- Hasse diagrams
- Least upper bounds and greatest lower bounds

Functions (Sec 4.1-4.4)

- Definition of a function
- Domain, codomain, image
- Identity functions, restrictions, inclusion maps
- Function composition
- Injectivity
- Surjectivity
- Left- and right-inverses
- Bijectivity
- (Two-way) inverses

Cardinality (Sec 5.1-5.5)

- Definition of $|A| = |B|$
- Finite cardinalities
- Countably infinite cardinality, \aleph_0 (and Hilbert's Hotel)
- Cantor's diagonalization argument
- Cardinality of the continuum, \mathfrak{c}
- Cantor's Theorem
- Comparing cardinalities with injections and surjections (Cantor-Schröder-Bernstein Theorem)

Algebra (Sec 6.1-6.5)

- Binary operations
- Definition of a group and of an abelian group
- Common examples of groups: $(\mathbb{Z}, +)$, $(\mathbb{Q} \setminus \{0\}, \cdot)$, $(\mathbb{Z}/m\mathbb{Z}, +)$, \mathfrak{S}_n , symmetry groups, etc.
- Cayley tables
- Subgroups
- Generators
- Order of a group, order of an element
- Group homomorphisms
- Isomorphism of groups

Practice Problems

- Let $f : A \rightarrow C$ and $g : B \rightarrow D$ be functions and let $h : A \times B \rightarrow C \times D$ be defined by $h(a, b) = (f(a), g(b))$.
 - Prove that if f and g are injective then h is injective.
 - Prove that if f and g are surjective then h is surjective.
- Determine if each function is injective and if it is surjective.
 - $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3$.
 - $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = |x|$.
 - $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x, y) = x - y$.
 - $f : \mathbb{Z}/6\mathbb{Z} \rightarrow \mathbb{Z}/6\mathbb{Z}$ defined by $f(\bar{x}) = \overline{2x + 1}$.
- Find a right-inverse for the quotient map $q : \mathbb{Z} \rightarrow \mathbb{Z}/3\mathbb{Z}$ defined by $q(x) = \bar{x}$.
- Prove that there exists a bijective function $f : \mathbb{R} \rightarrow \mathbb{R}^2$.
- Find the cardinality of each set and prove your answer.
 - $\mathcal{P}(\mathbb{Z} \times \{1, 2, 3\})$.
 - $\mathbb{Q} \cap [0, 1]$.
 - The symmetric group on four elements, \mathfrak{S}_4 .
- Let A be the set of functions from \mathbb{R} to \mathbb{Z} . Prove that A is uncountable.
- Prove that if A is an infinite set and B is a countably infinite set, then $|A \cup B| = |A|$.
- Prove that each pair of groups are not isomorphic.
 - $(\mathbb{Z}/5\mathbb{Z}, +)$ and $(\mathbb{Z}/6\mathbb{Z}, +)$
 - The symmetry group of a square and $(\mathbb{Z}/8\mathbb{Z}, +)$.
 - $(\mathbb{R} \setminus \{0\}, \cdot)$ and $(\mathbb{C} \setminus \{0\}, \cdot)$.
- Prove that $(\mathcal{P}(\mathbb{Z}), \Delta)$ is a group where Δ denotes the symmetric difference operation defined as $A\Delta B = (A \setminus B) \cup (B \setminus A)$.
- Write the Cayley table for $(\mathbb{Z}/5\mathbb{Z}, +)$.
- Find the order of each element of $(\mathbb{Z}/12\mathbb{Z}, +)$.
- Prove that the set $\{(x, x) \mid x \in \mathbb{R}\}$ is a subgroup of $(\mathbb{R}^2, +)$.
- Let $f : G \rightarrow H$ be a group homomorphism and let K be a subgroup of G . Prove that $\{f(k) \mid k \in K\}$ is a subgroup of H .
- Let R be a ring with multiplicative identity 1. For any $a \in R$ prove that $(-1) \cdot a = -a$.