MATH 150A Winter 2020 - **Problem Set 7**

due February 28

1. Find a presentation in terms of generators and relations for the following groups.
   (a) $\mathbb{Z} \times \mathbb{Z}$
   (b) $C_3 \times C_3$
   (c) S_3
   (d) A_4
2. The group $G = \langle x, y \mid xyx^{-1}y^{-1} \rangle$ is called a *free abelian group*. Prove a mapping property of this group: If u and v are elements of an abelian group A, there is a unique homomorphism $\varphi : G \to A$ such that $\varphi(x) = u$ and $\varphi(y) = v$.
3. Let F be the free group on $\{x, y\}$. Prove that the elements $u = x^2$, $v = y^2$, and $z = xy$ generate a subgroup isomorphic to the free group on $\{u, v, z\}$.
4. Let F be the free group on a nonempty set S with $|S| = k$. How many elements with reduced word of length n does F have?
5. (a) Prove that the additive group of $\mathbb{Q}$ is not finitely generated.
   (b) Prove that the multiplicative group $\mathbb{Q}^{\times}$ is not finitely generated.