MATH 150A Winter 2020 - **Problem Set 8**

due March 6

1. Draw the Cayley graph for each group and generating set.
   (a) C_{10} generated by $\{x\}$.
   (b) C_{10} generated by $\{x^2, x^5\}$.
   (c) A_4 generated by $\{(1\ 2\ 3), (2\ 3\ 4)\}$.
   (d) $C_2 \times C_2 \times C_2$ generated by $\{(x,1,1), (1,x,1), (1,1,x)\}$.

2. Let G be a group generated by S and H a subgroup of G generated by $T \subseteq S$. Prove that H is normal in G if and only if all edges labelled by elements of T are loops in the Schreier coset graph of H in G with generating set S.

3. Given two elements of the lamplighter group

$$g = (n, (\ldots, l_{-1}, l_0, l_1, \ldots)),$$

$$h = (m, (\ldots, k_{-1}, k_0, k_1, \ldots)),$$

   how can one determine if they are conjugates?

4. The *infinite dihedral group* D_∞ is a subgroup of permutations of the integers generated by $f(n) = -n$ and $g(n) = 1 - n$, which reflect the integer number line over the point 0 and 1/2 respectively.
   (a) Give a presentation of D_∞.
   (b) Demonstrate a surjective homomorphism to each finite dihedral group $\varphi : D_\infty \to D_n$ for $n \geq 3$.

5. Use the Todd-Coxeter algorithm to analyze the group generated $\{x, y\}$ with the following relations. Determine the order of the group and identify the group if you can.
   (a) $x^2 = 1$, $y^2 = 1$, $xyx = yxy$,
   (b) $x^3 = 1$, $y^3 = 1$, $xyx = yxy$,
   (c) $x^4 = 1$, $y^2 = 1$, $xyx = yxy$,
   (d) $x^4 = 1$, $y^4 = 1$, $x^2y^2 = 1$,
   (e) $x^3 = 1$, $y^2 = 1$, $yxyxy = 1$,
   (f) $x^3 = 1$, $y^3 = 1$, $yxyxy = 1$.